Stochastic modeling and financial applications

VPSMS 2018

# Book of Abstracts

June 11–14, 2018
Verona, Italy

# Contents

# Monday, June 11th, 2018

## Quenched mass transport of particles towards a target

Monday
Jun 11
09h40-
10h20

Idris Kharroubi
*UPMC*

We consider the stochastic target problem of finding the collection of initial laws of a mean-field stochastic differential equation such that we can control its evolution to ensure that it reaches a prescribed set of terminal probability distributions, at a fixed time horizon. Here, laws are considered conditionally to the path of the Brownian motion that drives the system. We establish a version of the geometric dynamic programming principle for the associated reachability sets and prove that the corresponding value function is a viscosity solution of a geometric partial differential equation. This provides a characterization of the initial masses that can be almost surely transported towards a given target, along the paths of a stochastic differential equation. This talk is based on a joint work with Bruno Bouchard and Boualem Djehiche.

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## Zero-sum stochastic differential games of generalized McKean-Vlasov type

Monday
Jun 11
11h00-
11h40

Andrea Cosso
*Università di Bologna*

We discuss zero-sum stochastic differential games where the state dynamics of the two players is governed by a generalized McKean-Vlasov (or mean-field) stochastic differential equation in which the distribution of both state and controls of each player appears in the drift and diffusion coefficients, as well as in the running and terminal payoff functions. We address such a problem using the dynamic programming approach (in other words, the time-consistency approach). In particular, we state the dynamic programming principle (DPP) in this general setting. We also introduce the upper and lower Master Bellman-Isaacs equations related to this stochastic differential game. Our results extend the seminal work of Fleming and Souganidis to the McKean-Vlasov setting. This is a joint work with Huyn Pham.

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## Kyle-Backs model with a random horizon

Monday
Jun 11
11h40-
12h20

Giulia Di Nunno
*University of Oslo*

We study the continuous-time version of Kyle (1985) developed by Back (1992). In Backs model there is asymmetric information in the market in the sense that there is an insider having information on the real value of the asset. We extend this model by assuming that the fundamental value evolves with time and that it is announced at a future random time. First we consider the case when the release time of information is predictable to the insider and then when it is not. Our goal is to study the structure of equilibrium, which is described by the optimal insider strategy and the competitive market prices given by the market makers. We provide necessary and sufficient conditions for the optimal insider strategy under general dynamics for the asset demands. Moreover, we study the behaviour of the price pressure and the market efficiency. In particular we find that when the random time is not predictable, there can be equilibrium without market efficiency. Furthermore, for the two cases of release time and for classes of pricing rules, we provide a characterization of the equilibrium. This is joint work with Jos Manuel Corcuera, University of Barcelona.

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## Self-Enforcing Risksharing Arrangements in a Continuous-Time Endowment Economy

Monday
Jun 11
14h20-
15h00

Mihalis Zervos
*London School of Economics*

We characterise efficient risksharing under two-sided limited commitment in a continuous-time endowment economy. We take a dual approach to the problem based on the Kuhn-Tucker multipliers on the participation constraints, and establish a strong duality result in a general setting. In an application, agents have the same time-separable preferences, aggregate endowment is constant, and endowment shares are driven by a mean-reverting diffusion process. The value function for the dual problem is homogeneous and the relevant co-state variable is a 'modified' relative Pareto-Negishi weight, which determines the consumption allocation. We analyse the HJB equation associated with the singular control problem and solve for the free boundaries that determine regions of the state space where the participation constraints are binding. In particular, we show that, for some parameter values, perfect risksharing can be sustained despite limited commitment.

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## Limited Liability in the Continuous-time Principal-Agent problem with CARA

Monday
Jun 13
15h00-
15h40

Stéphane Villeneuve

*Université Toulouse 1*

In this paper, we examine optimal managerial compensation in a continuous-time principal- agent model in which both principal and agent have exponential utilities and are protected by limited liability. Limited liability imposes strong constraints on the contract: it cannot impose negative payments to the agent, and the principals promise keeping is restricted to cash within the firm. By means of the theory of BSDE, we embed the principal-agent problem into the class of Markovian stochastic control problems under state constraint and characterize the principal value function in terms of HJB equation.

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## Sensitivity Analysis for Marked Hawkes Processes - Application to CLO Pricing

Monday
Jun 11
16h20-
17h00

Guillaume Bernis

*Ostrum AM*

This paper deals with a model for pricing Collateralized Loan Obligations, where the underlying credit risk is driven by a marked Hawkes process, involving both clustering effects on defaults and random recovery rates. We provide a sensitivity analysis of the CLO price with respect to the parameters of the Hawkes process using a change of probability and a variational approach. We also provide a simplified version of the model where the intensity of the Hawkes process is taken as the instantaneous default rate. In this setting, we give a moment-based formula for the expected survival probability. This is a joint work with Kaouther Salhi and Simone Scotti.

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## A Score-Driven Conditional Correlation Model for Noisy and Asynchronous Data: An Application to High-frequency Covariance Dynamics

Monday
Jun 11
17h00-
17h40

Fabrizio Lillo

*Università di Bologna*

We propose a new multivariate conditional correlation model able to deal with data featuring both observational noise and asynchronicity. When modelling high-frequency multivariate financial time-series, the presence of both problems and the requirement for positive-definite estimates makes the estimation and forecast of the intraday dynamics of conditional covariance matrices particularly difficult. Our approach tackles all these challenging tasks within a new Gaussian state-space model with score-driven time-varying parameters that can be estimated using standard maximum likelihood methods. Similarly to DCC models, large dimensionality is

handled by separating the estimation of correlations from individual volatilities. As an interesting outcome of this approach, intra-day patterns are recovered without the need of any cross-sectional averaging, allowing, for instance, to estimate the real-time response of the market covariances to macro-news announcements.

# Tuesday, June 12th, 2018

## Infinite dimensional polynomial processes

Christa Cuchiero
*University of Vienna*

Tuesday
Jun 12
09h00-
09h40

Motivated from high and infinite dimensional problems in mathematical finance, we consider in- finite dimensional polynomial processes taking values in certain space of measures or functions. We have two concrete applications in mind: first, modelling high or even potentially infinite dimensional financial markets in a tractable and robust way, and second analysing stochastic Volterra processes, which recently gained popularity through rough volatility models and ambit processes. The first question leads to probability measure valued polynomial diffusions and the second one to Markovian lifts of polynomial Volterra processes. For both cases we provide existence and uniqueness results and a moment formula. .

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## Alpha-Heston model

Chunhua Ma
*Nankai University*

Tuesday
Jun 12
09h40-
10h20

We introduce a new stock or exchange rate model, called the $\alpha$-Heston model, which is a natural extension of the standard Heston model (1993) by adding a jump part driven by a $\alpha$-stable Lévy process in the variance process in the spirit of Dawson and Li (2006), Fu and Li (2010) and Li and Ma (2015). We deduce an explicit expression of the Laplace transform by using the fact that the model belongs to the class of affine processes, and mainly examine the impact of jumps on the implied volatility smile for the option written on the variance and VIX process. In particular, we characterize the distribution tails of variance process showing that the right tails is controlled by the parameter $\alpha$ whereas the left one depends only on CIR usual parameters. Furthermore we provide a new decomposition of clustering effect, splitting large jumps into immigration and births, to give a intuitive description of self-exciting structure of jumps.

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## Quantization goes polynomial

Tuesday Jun 12 11h00-11h40

Giorgia Callegaro

*Universitá di Padova*

Quantization algorithms have been recently successfully adopted in option pricing problems to speed up Monte Carlo simulations thanks to the high convergence rate of the numerical approximation. In particular, recursive marginal quantization has been proven a flexible and versatile tool when applied to stochastic volatility processes. In this paper we apply for the first time these techniques to the family of polynomial processes, by exploiting, whenever possible, their peculiar properties. We derive theoretical results to assess the approximation errors, and we describe in numerical examples practical tools for fast exotic option pricing.

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## Sampling of probability measures in the convex order and approximation of Martingale Optimal Transport problems

Tuesday Jun 12 11h40-12h20

Aurélien Alfonsi

*CERMICS - Ecole Nationale des Ponts et Chaussées*

In this work, motivated by the approximation of Martingale Optimal Transport problems, we are interested in sampling methods preserving the convex order for two probability measures $\mu$ and $\nu$ on $\mathbb{R}^d$, with $\nu$ dominating $\mu$ . When $(X_i)_{1\leq i\leq I}$ (resp. $(Y_j)_{1\leq j\leq J}$) are independent and identically distributed according $\mu$ (resp. $\nu$), in general $\mu_I = \frac{1}{I}\sum_{i=1}^I \delta_{X_i}$ and $\nu_J = \frac{1}{J}\sum_{j=1}^J \delta_{Y_j}$ are not rankable for the convex order. We investigate modifications of $\mu_I$ (resp. $\nu_J$) smaller than $\nu_J$ (resp. greater than $\mu_I$) in the convex order and weakly converging to $\mu$ (resp. $\nu$) as $I, J \to \infty$. We first consider the one dimensional case $d = 1$, where, according to Kertz and Rösler (1992), the set of probability measures with a finite first order moment is a lattice for the increasing and the decreasing convex orders. Given $\mu$ and $\nu$ in this set, we define $\mu \vee \nu$ (resp. $\mu \wedge \nu$) as the supremum (resp. infimum) of $\mu$ and $\nu$ for the decreasing convex order when $\int_{\mathbb{R}} x\mu(dx) \leq \int_{\mathbb{R}} x\nu(dx)$ and for the increasing convex order otherwise. This way, $\mu \vee \nu$ (resp. $\mu \wedge \nu$) is greater then $\mu$ (resp. smaller than $\nu$) in the convex order. We give efficient algorithms permitting to compute $\mu \vee \nu$ and $\mu \wedge \nu$ (and therefore $\mu_I \vee \nu_J$ and $\mu_I \wedge \nu_J$) when $\mu$ and $\nu$ are convex combinations of Dirac masses. In general dimension, when $\mu$ and $\nu$ have finite moments of order $\rho \geq 1$, we define the projection $\mu \curlywedge_\rho \nu$ (resp. $\mu \curlyvee_\rho \nu$) of $\mu$ (resp. $\nu$) on the set of probability measures dominated by $\nu$ (resp. larger than $\mu$) in the convex order for the Wasserstein distance with index $\rho$. When $\rho = 2$, $\mu_I \curlywedge_2 \nu_J$ can be computed efficiently by solving a quadratic optimization problem with linear constraints. It turns out that, in dimension $d = 1$, the projections do not depend on $\rho$ and their quantile functions are explicit in terms of those of $\mu$ and $\nu$, which leads to efficient algorithms for convex combinations of Dirac masses. Last, we illustrate by numerical experiments the resulting sampling methods that preserve the convex order and their application to approximate Martingale Optimal Transport problems.

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# Wednesday, June 13th, 2018

## On continuous time contract theory

Wednesday
Jun 13
09h00-
09h40

Nizar Touzi
*Ecole Polytechnique*

The Principal-Agent problem is the corner stone for the modelling of optimal incentive schemes to account for moral hazard in economics. Its continuous time formulation is a nonzero sum Stackelberg stochastic differential game problem, and has important applications in modern electronic economic systems. A systematic method for solving this class of problems is derived by crucial use the backward SDEs and their second order extension. The infinite horizon setting requires an extension of such backward SDEs to the random horizon setting.

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## Optimal dividend and investment policy with debt covenants

Wednesday
Jun 13
11h00-
11h40

Etienne Chevalier
*Université d'Evry*

We consider a firm that holds a certain amount of debt to which is associated a financial-ratio covenant between the creditors and the firm. Under this debt covenant, the firm may be audited at any time and must furnish financial statements to prove that its debt to total assets ratio is less than one. If not, the firm is given a grace period during which it can inject more capital to correct the situation. When its ratio is less than one, it can also pay out dividends. Under this setup, we consider an optimal control problem in which the value of the assets is controlled by the capital injections and the dividends payouts. This gives rise to a system of variational inequalities to which there exists a unique viscosity solution..

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## Mean reflected stochastic differential equations with jumps

Wednesday
Jun 13
11h00-
11h40

Céline Labart

*Université de Savoie*

We study reflected Stochastic Differential Equations with jumps when the constraint is not on the paths of the solution but acts on the law of the solution. This type of reflected equations have been introduced recently by Briand, Elie and Hu in the context of BSDEs, when no jumps occur. We also present a numerical scheme based on particle systems to approximate these reflected SDEs.

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## Equilibrium Model for Virtual Currencies

Wednesday
Jun 13
11h40-
12h20

Alexandre Roch

*UQAM*

Virtual currencies are assets that intrinsically have no value but that serve as a medium of exchange. Furthermore, their utility for commercial transactions (the amount of goods exchanged with it) pales in comparison to the amount of speculation associated to it. In this talk, I will present an equilibrium model for cryptocurrencies (e.g., bitcoin) in which consumers trade with producers, and speculators have an impact on the price of the currency. The equilibrium, constructed from a sequence of approximate equilibria, is shown to be represented by a representative consumer and a representative speculator.

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## Pricing path-dependent Bermudan options

Wednesday
Jun 13
14h20-
15h00

Jérôme Lelong

*ENSIMAG*

We present an algorithm based on the policy iteration approach to price path dependent Bermudan options. The least square method proposed by Longstaff and Schwartz requires a Markovian framework, which is not available for real path dependent options. We will present an alternative approach to approximate conditional expectations with respect to the whole past. We will discuss how to build upper confidence intervals for our approach. Finally, we will conclude with some numerical tests of our algorithm showing the impressive efficiency of its parallel implementation.

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## Uncertainty Quantification for XVA Applications

Wednesday
Jun 13
15h00-
15h40

Stéphane Crépey

*LaMME-Université Evry-CNRS-Université Paris-Saclay*

- Stochastic Approximation (SA) Method and Applications;
- Uncertainty Quantification for SA Limits;
- The USA Algorithm;
- Numerical Study;
- XVA Applications.

Joint work with Gersende Fort, Emmanuel Gobet, and Uladzislau Stazhynski.

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## Stability and other results about path dependent calculus

Wednesday
Jun 13
16h20-
17h00

Cristina Di Girolami

*Univeritá di Chieti-Pescara*

This talk develops some aspects of stochastic calculus via regularization for path dependent random variables. After some brief reminds on stochastic calculus in a general Banach space B, some stability results for processes admitting this kind of quadratic variation are presented. Main interest will be devoted to the case when B is the space of real continuous functions defined on [-T,0], T¿0 and the process is the path of a continuous stochastic process with finite quadratic variation (behind semimartingale processes of course, for instance Dirichlet, weak Dirichlet, ecc...). We show that it is possible to represent a large class of path-dependent real valued random variable h as a real number plus a real forward integral in a semiexplicite form. This representation result of h makes use of a functional solving a path dependent infinite dimensional partial differential equation of Kolmogorov type. Two recent general existence results of its classical solutions related to different classes of final conditions will be presented. The decomposition result generalizes, in some cases, the well known Clark-Ocone formula which is true when X is the standard Brownian motion W. Some examples will be given explicitly developed and discussed.

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## Affine Volterra processes

Sergio Pulido

*ENSIIE*

Wednesday
Jun 13
17h00-
17h40

Motivated by recent advances in rough volatility modeling, we introduce affine Volterra processes, defined as solutions of certain stochastic convolution equations with affine coefficients. Classical affine diffusions constitute a special case, but affine Volterra processes are neither semimartingales, nor Markov processes in general. Nonetheless, their Fourier-Laplace functionals admit exponential-affine representations in terms of solutions of associated deterministic integral equations, extending the well-known Riccati equations for classical affine diffusions. Our findings generalize and clarify recent results in the literature on rough volatility. This is joint work with Eduardo Abi Jaber and Martin Larsson.

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# Thursday, June 14th, 2018

## Optimal make take fees for market making regulation

Thursday Jun 14 09h00-09h40

Thibaut Mastrolia

*Ecole Polytechnique*

We consider an exchange who wishes to set suitable make-take fees to attract liquidity on its platform. Using a principal-agent approach, we are able to describe in quasi-explicit form the optimal contract to propose to a market maker. This contract depends essentially on the market maker inventory trajectory and on the volatility of the asset. We also provide the optimal quotes that should be displayed by the market maker. The simplicity of our formulas allows us to analyze in details the effects of optimal contracting with an exchange, compared to a situation without contract. We show in particular that it leads to higher quality liquidity and lower trading costs for investors. Joint work with Omar El Euch, Mathieu Rosenbaum and Nizar Touzi.

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## Stochastic Analysis of Investment Products with Capital Protection

Thursday Jun 14 09h40-10h20

Kai Wallbaum

*Allianz*

Designing an investment product with capital protection is a challenging task in the current market environment. Low interest rates and high volatility levels negatively affect key parameters of such investment solutions. One way to design an investment product with capital protection is to use a dynamic asset allocation mechanism as an underlying asset for a financial option embedded in such a product. We considered especially protection strategies (e.g. CPPI with guaranteed minimum equity exposure) and so-called Target Volatility strategies and performed a numerical study of options linked to such asset allocation strategies in a stochastic market environment.

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# Optimal control under controlled loss constraints via reachability approach and compactification

Thursday
Jun 14
11h00-
11h40

Athena Picarelli

*Imperial College London*

We study optimal control problems under controlled loss constraints at several fixed dates. It is well known that for such problems the characterization of the value function by a Hamilton-Jacobi-Bellman equation requires additional strong assumptions involving an interplay between the set of constraints and the dynamics of the controlled system. To treat the problem in absence of these assumptions we first translate it into a state-constrained stochastic target problem and then apply a level-set approach to describe the reachable set. The main advantage of our approach is that it allows us to easily handle the state constraints by an exact penalization. However, this target problem involves a new set of control variables that are unbounded. A "compactification" of the problem is then performed.

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## Organizing committee

- Francesco Giuseppe Cordoni
- Luca Di Persio
- Romuald Elie
- Vathana Ly Vath
- Simone Scotti
- Carlo Sgarra

# Author Index